

سنة ثالثة اهتزازات

حل المدين 1: إيجاد المعادلات التفاضلية باستعمال طريقة الحجوم

$$\sum \mathcal{U}_b(\vec{F}_{ext}) = \ddot{\Theta} \quad (1)$$

هناك ثلاث حجوم:

$$\mathcal{U}_b(\vec{P}) = \vec{OB} \wedge \vec{P} = -mgL \sin \theta$$

$$\mathcal{U} = -mgL \theta$$

$$\sin \theta \approx \theta \quad \theta \text{ صغير}$$

$$\mathcal{U}_b(\vec{F}_1) = \vec{OA} \wedge \vec{F}_1$$

$$= -aF_1 \cos \theta$$

$$= -a(k_1 x) \cos \theta$$

$$= -k_1^2 a^2 \theta \quad (1)$$

$$\mathcal{U}_b(\vec{F}_2) = \vec{OA} \wedge \vec{F}_2 = -k_2^2 a^2 \theta \quad (1)$$

$$J = mL^2 \quad (1)$$

$$\mathcal{U}_b(\vec{P}) + \mathcal{U}_b(\vec{F}_1) + \mathcal{U}_b(\vec{F}_2) = J \ddot{\Theta}$$

$$-mgL \theta - k_1^2 a^2 \theta - k_2^2 a^2 \theta = mL^2 \ddot{\Theta}$$

$$\ddot{\Theta} + \frac{mgL + a^2 \theta (k_1 + k_2)}{mL^2} \theta = 0 \quad (1)$$

$$L = T - U$$

طريقة لاغرانج

$$\text{الطاقة الحركية: } T = \frac{1}{2} \dot{\theta}^2 = \frac{1}{2} (mL^2) \dot{\theta}^2 = \boxed{\frac{1}{2} mL^2 \dot{\theta}^2}$$

الطاقة الكامنة: $U = ?$

① ✓

$$U_p = mgh = mgL(1 - \cos \theta)$$

$$U_k = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 x^2 = \frac{1}{2} k_1 (a\theta)^2 + \frac{1}{2} k_2 (a\theta)^2$$

$$x = a\theta$$

$$= \boxed{\frac{1}{2} a^2 (k_1 + k_2) \theta^2}$$

① ✓

$$L = \frac{1}{2} mL^2 \dot{\theta}^2 - mgL(1 - \cos \theta) - \frac{1}{2} a^2 (k_1 + k_2) \theta^2$$

$$\frac{\partial L}{\partial t} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = mL^2 \ddot{\theta} - (mgL \theta - a(k_1 + k_2) \theta) = 0$$

✓

$$mL^2 \ddot{\theta} + (mgL + a(k_1 + k_2)) \theta = 0$$

①

$$\boxed{\ddot{\theta} + \left(\frac{mgL + a(k_1 + k_2)}{mL^2} \right) \theta = 0}$$

حل التمرين 2.2

(1) المعادلات التفاضلية 2

$$L = T - U = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k_{eq} x^2$$

$$k_{eq} = k_1 + k_2 + k_3 = 3k \quad (1)$$

$$\frac{dL}{dt} \left(\frac{dL}{dx} \right) - \frac{dL}{dx} = 0 \Rightarrow \left[\ddot{x} + \frac{k_{eq}}{m} x = 0 \right]$$

$$\left[\ddot{x} + \omega_0^2 x = 0 \right]$$

$$T = \frac{1}{3} s \quad \text{؟ } k \text{ مطلوب (2)}$$

$$\omega_0^2 = \frac{k_{eq}}{m} \Rightarrow k_{eq} = \omega_0^2 m = \left(\frac{2\pi}{T} \right)^2 m =$$

$$3k = \left(\frac{2\pi}{T} \right)^2 m \quad (1)$$

$$\Rightarrow k = \left(\frac{2\pi}{T} \right)^2 \frac{m}{3} = \left[47.37 \times 10^4 \text{ N/m} \right]$$

$$(1) \quad \omega_0 = \sqrt{\frac{k_{eq}}{M+m}} = 5.68 \text{ rad/s} \quad (3)$$

$$\Rightarrow T = \frac{2\pi}{\omega_0} = \frac{2(3.14)}{5.68} = 1.1 s$$

$$\frac{dL}{dt} \left(\frac{dL}{dx} \right) - \frac{dL}{dx} = -b\dot{x} \quad (4)$$

$$L = T - U = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k_{eq} x^2$$

$$m\ddot{x} + b\dot{x} + k_{eq}x = 0$$

$$\ddot{x} + \frac{b_{eq}}{m}\dot{x} + \frac{k_{eq}}{m}x = 0$$

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0$$

$$\delta = \frac{b_{eq}}{2m}, \quad \omega_0 = \sqrt{\frac{k_{eq}}{m}}, \quad b = b_1 + b_2$$

5) معطى: $\omega_0^2 = 1$ ، $\delta = 0.22$

$$x(t) = c e^{-\delta t} (\cos(\omega_0 t + \varphi))$$

المعادلة التفاضلية

$$A(t) = c e^{-\delta t}$$

$$A(t+T) = c e^{-\delta(t+T)}$$

بعد دور

$$\frac{A(t)}{A(t+T)} = e^{-\delta T} = \frac{1}{4} \Rightarrow \delta T = \log 4$$

$$\zeta = \frac{\delta}{\omega_0}, \quad T = \frac{2\pi}{\omega_a} = \frac{2\pi}{\omega_0 \sqrt{\omega_0^2 - \delta^2}} = \frac{2\pi}{\omega_0 \sqrt{1 - \zeta^2}}$$

$$T = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}} \Rightarrow \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}} = \log 4$$

$$\Rightarrow \zeta = 0.22 \quad \omega_0 = \frac{2\pi}{T} = 18.85 \text{ rad/s}$$

$$\delta = \zeta \omega_0 = 4.15 \text{ s}^{-1}, \quad b = 2\delta m = 33200 \text{ N.s.m}^{-1}$$

$$Q = \frac{\omega_0}{2\delta} = \frac{1}{2\zeta} = 2.27$$

لاحصل على شامد خرج لابد ان يكون $\delta = \omega_0$

$$\omega_0 = 5168 \text{ rad/s}$$

1 $\Rightarrow \delta' = \frac{b'}{2(m+M)} \Rightarrow b' = \delta' 2(m+M) = 499840 \text{ N/s}$
8.1m